Mann-Kendall Test

The Mann-Kendall Trend Test (maybe called the MK test) is used to analyze data collected over time for consistently increasing or decreasing trends. The Mann-Kendall (MK) statistical test has been widely applied in the trend detection of the hydrometeorological time series. It is a non-parametric test, which means It's **distribution-free**, but assumes **independent observations** and **no serial correlation**. If your data do follow a normal distribution, you can run simple linear regression instead.

Mann-Kendall test in Data-tool has a simple process and you can get the items S and Z based on the below formula. For interpretation of results follow the papers like this paper. For doing this test you can follow the below video.

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$$S = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \operatorname{sgn}\left(x_j - x_i\right). \quad (1)$$

In Eq. (1), n is the number of data points, x_i and x_j are the data values in time series i and j (j > i), respectively and in Eq. (2), sgn ($x_j - x_i$) is the sign function as

$$\operatorname{sgn}\left(x_j - x_i\right) = \begin{cases} +1, \text{if } \left(x_j - x_i\right) > 0 \\ 0, \text{if } \left(x_j - x_i\right) = 0 \\ -1, \text{if } \left(x_j - x_i\right) < 0 \end{cases}. \quad (2)$$

The variance is computed as

$$V(S) = \frac{n(n-1)(2n+5) - \sum_{k=1}^{m} t_k \, (t_k - 1)\,(2t_k + 5)}{18}. \quad (3)$$

In Eq. 3, n is the number of data points, m is the number of tied groups, and t_k denotes the number of ties of extent k. A tied group is a set of sample data having the same value. In cases where the sample size n > 10, the standard normal test statistic Z_S is computed using Eq. (4):

$$Z_S = \begin{cases} \frac{S-1}{\sqrt{V(S)}}, & \text{if } S > 0 \\ 0, & \text{if } S = 0 \\ \frac{S+1}{\sqrt{V(S)}}, & \text{if } S < 0 \end{cases}. \quad (4)$$

Trend Test using Hypothesis Test for Regression Slope

It looks like you're describing the t-test for the slope coefficient (B) in a simple linear regression model, where Y is the dependent variable, X is the independent variable, B is the slope coefficient (also denoted as β_1), and A is the intercept (also denoted as β_0).

Let's break down the steps based on the provided formulas:

1. Regression Model: The regression model you're considering is:

$Y = B.X + A$

Where:

- Y is the dependent variable.
- X is the independent variable.
- B is the slope coefficient.
- A is the intercept.

2. Null Hypothesis (H_0): The null hypothesis states that there is no relationship between the independent variable and the dependent variable. Specifically, it suggests that the true slope coefficient (B_{true}) is equal to zero.

$H_0: B_{true} = 0$

3. Test Statistic: The test statistic (t) for the t-test of the slope coefficient is calculated as:

$$t = \frac{B_{hat} - B_{true}}{SE}$$

Where:

- B_{hat} is the estimated slope coefficient from the regression model.

- B_{true} is the hypothesized true slope coefficient (in this case, 0 means no trend).
- SE is the standard error of the slope coefficient.

4. Degrees of Freedom: The degrees of freedom (ν) for the t-test of the regression slope is typically n - 2, where n is the sample size and 2 accounts for the two parameters estimated in the regression model (intercept and slope).

5. Critical Value: To determine whether to reject the null hypothesis, you compare the absolute value of the calculated t-statistic to the critical value from the t-distribution with n - 2 degrees of freedom at a chosen significance level (α).

x = T.Inv(p, ν)

Where:

- p is the probability (usually chosen significance level).
- ν is the degrees of freedom.

If the absolute value of the calculated t-statistic is greater than the critical value, you reject the null hypothesis, indicating a significant relationship between the variables. Otherwise, you fail to reject the null hypothesis.

These steps outline the t-test for the slope coefficient in a simple linear regression model. It's a fundamental tool for assessing the significance of the relationship between variables in regression analysis.

WHITE's Trend Test

White’s Test for Heteroskedasticity, also known as White’s General Test or White’s Trend Test, is a statistical procedure used to identify heteroscedasticity in regression analysis. Heteroscedasticity occurs when the variance of the errors (residuals) in a regression model varies across different levels of the independent variables.

Key points about White’s Test:

White’s test is **not usually described as a “trend test.”** It's for **heteroskedasticity**, not time series trend.

Purpose: Detecting heteroscedasticity, which violates the assumption of constant variance in the error terms.

Flexibility: White’s Test is a general test that can identify various patterns of non-constant variance, including nonlinear relationships between predictors and residual variance.

Robustness: It is robust and widely used in econometrics and other fields.

Here's the breakdown of the formulas for White's Test:

1. Regression Model:

- The regression model is represented as Y = BX + res, where:
    - Y is the dependent variable (observations).

- X is the independent variable.
- B is the coefficient vector (including the intercept).
- res represents the residuals.

2. Estimation of Coefficients (Bhat):

   - The coefficients B are estimated using ordinary least squares (OLS) regression.

3. Squared Residuals (RES^2):

   - Calculate the squared residuals RES^2 as the squared deviations of the observed values from the predicted values.

4. Auxiliary Regression:

   - A quadratic regression is applied to the squared residuals: $RES^2 = A * X_{res} + E_{res}$, where:

     - $X_{res} = [1, X, X^2]$ is the design matrix including a column of ones, X, and X^2.
     - A is the coefficient vector for auxiliary regression.
     - E_{res} represents the residuals of the auxiliary regression.

5. Calculation of R^2:

- Calculate the coefficient of determination (R^2) for the residuals. This is done by comparing the explained sum of squares (ESS) to the total sum of squares (TSS), where:

$$R^2 = \frac{ESS_r}{TSS_r}$$

- ESS_r is the explained sum of squares (Regression sum of squares).
- TSS_r is the total sum of squares (proportional to the variance of the data).

6. White Test Statistic (LM):

- Calculate the White test statistic LM, which is a scaled version of R^2: $LM = n * R^2$, where n is the number of observations.

7. Chi-Square Test:

- Conduct a chi-square test using the White test statistic.
- Determine the critical value z from the chi-square distribution with degrees of freedom dof equal to the number of parameters in the auxiliary regression minus one.
- Calculate the p-value (p_v) based on the chi-square distribution.
- Compare the White test statistic to the critical value to determine whether to reject the null hypothesis of homoscedasticity.